THE EFFICIENCY OF THE EUROPEAN FOREIGN EXCHANGE MARKET: A Cointegration Analysis with High Frequency Data

JOSE MIGUEL GREDILLA [†]
Universidad Técnica Federico Santa María
Departamento de Industrias, Santiago, Chile

MANUEL MADRID-ARIS[†]
Universidad Técnica Federico Santa María
Departamento de Industrias, Santiago, Chile

MARCELO VILLENA
Universidad Adolfo Ibáñez
Facultad de Ciencias y Tecnología, Santiago, Chile

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SUMMARY

The main objective of this paper is to analyze the efficiency of the European currency markets after the Euro came into force. Using econometric techniques the existence of a long and short-term relationship between the spot rate of the Euro with high frecuency data is contrasted with other European currencies, such as the Danish Crown, the Norwegian Crown, the Sterling Pound, and the Swiss Franc. By means of standard cointegration tests the existence of a long term relationship between the Danish Crown and the Euro, and between the Norwegian Crown and the Euro, has been established, and so damaging the hypothesis of market efficiency. This relationship is maintained when high volumes of these currencies are traded on the international market. In addition to this, cointegration analyses were carried out for certain times of day, establishing the presence of cointegration for the same currencies between 8:40 and 9:30, and between 9:50 and 10:40 (Eastern Standard Time). Finally, the predictive ability of different VAR models with one month and one week time horizons are analyzed for the Danish and Norwegian Crown with regard to the Euro.

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[†] The authors would appreciate commentaries and correspondence sent to Manuel Madrid (manuel.madrid@usm.cl).

Introduction

The analysis of the efficiency of different currency markets is one of international finance's very active research areas, where countless studies have been developed with the aim of refuting the market efficiency theories originally proposed by Samuelson (1965), Roberts (1967) and Fama (1970), among others. In this field the work of Granger (1986) stands out, who pointed out that two financial assets in an efficient market cannot be cointegrated, for if they were, the price of one asset could be used to predict the price of the other. [1]

In spite of this interest, a great majority of these works have focused on contrasting the efficiency of finance markets with low frequency observations that is quarterly, monthly, or daily. One interesting exception is the study prepared by Engle and Russell (2004), who make a very complete revision with respect to the high frequency financial data, which is currently available.

The main objective of this study is to analyze the efficiency hypothesis in European foreign exchange markets for those countries that have not adopted the Euro as their sole currency, in light of high frequency data[2]. Specifically, the presence of cointegration among different exchange rates in a highly developed and automated market will be analyzed, taking into account different criteria such as: i) high volumes traded, and ii) different daily times. On the other hand, prediction models will be developed in order to verify the possibility of obtaining higher than average profits from these foreign currencies, utilizing buying and selling strategies.

This study is divided into five sections. The first section presents the revision of the literature showing similar studies carried out in this area and their main results. The second section shows the methodology used in the study. In the third section the data used are described, and the main findings and results of the cointegration analysis are set forth. In the fourth section VAR models are presented for those foreign currencies, which demonstrate cointegration relationships with the Euro, and their predictive capability analyzed. Finally the conclusions and recommendations of this study are stated.

1.- Literature Review

A great amount of research exists which makes reference to market efficiency and performance. In general, varied international studies have been developed regarding the topic of cointegration among different financial instruments, with differing results.

Fama (1970), a classic work in this area, points out that a market in which prices completely reflect the available information, is denominated *"efficient"*. In his research he characterizes

[1] It is said that two or more price series are cointegrated if they move conjointly over time and the differences between them are stable, even when they are not stationary. From here cointegration reflects the presence of a long term balance towards which the economic system converges in time.

[2] Switzerland, Norway, and Great Britain are not part of the European Union, however, Denmark is a member of the community but has not adopted the Euro as its official currency.

three forms of market efficiency; the "weak form", which contemplates the historical price information, the "semi-strong form", which in addition considers public information, and the "strong form", which also includes the monopolistic access that agents have to certain private information. These three forms of market efficiency make up the known hypothesis of the efficient market (EHM). In this publication tests are carried out to contrast the hypothesis of the distinct forms of efficiency, using daily price data from thirty stocks from the United States, from the end of 1957 to September 1962. Concluding the existence of efficiency in the three above-mentioned forms.

Engle and Granger (1987) develop procedures using estimation, tests, and empirical examples to establish long term relationships among time series, which is known as cointegration; besides devising error correction models (ECM), permitting the short term dynamics of these series to be represented. To this end a methodology for contrasting the hypothesis of cointegration between two series is developed. These procedures use unit root tests to verify the non-stationary of the series and the corresponding stationary of their linear combinations[3], and so confirm the existence of cointegration. Moreover, empirical tests are carried out to prove the performance of the tests in practice, using quarterly data of personal consumption and income (quarter I - 1947 until quarter II - 1981), demonstrating the existence of cointegration between these two variables. In the following some more current studies related to the present research are examined.

Zivot (1998) investigates the relationship between cointegration models related to the current spot rate (S_t) of a foreign currency and its corresponding forward rate (f_t), and models that use the future spot rate (S_{t+1}) and the current forward rate (f_t). Using monthly data of the Sterling Pound, the Yen and the Canadian Dollar with respect to the American Dollar for his evidence and model formulation. The study's main conclusion shows a relationship with which the formulation of a simple cointegration model between the current forward and spot rates capture more readily those stylized facts of exchange rates and their performance, in comparison with models that use lagged spot rates in a period.

Dutt and Ghosh (1999) examine the weak and strong efficiency forms in the foreign currency market using a procedure alternate to that proposed by Engle and Granger. To do so they perform the traditional test of Dickey-Fuller, for certain currencies such as the Danish Crown, the French Franc, the German Mark, the Belgian Franc, the Italian Lire, and the Dutch Florin, comparing the foreign currencies' forward and spot rates[4], measured with respect to the American Dollar and using monthly data. Subsequently the tests proposed by Harris and Inder (1994) are carried out for the same currencies. For both methods the same result is obtained: the non-existence of cointegration between the currencies' spot rates and their corresponding forward rates, which is theoretically interpreted as inefficiency in the long term foreign currency market.

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3 Let $\{X_t\}$ and $\{Y_t\}$ be two series of time that, after differentiating them d times they become stationary, then both $\{X_t\}$ and $\{Y_t\}$ are integrated of the order d, this is I(d). Then $\mu_t=Y_t - \beta \cdot X_t$ should be I(d - b), with $b > 0$, so that both series are cointegrated.

4 It is said that when there is cointegration between the spot and forward rates of a given currency, this is interpreted as the corresponding market being efficient in the long term.

Kellard, Newbold and Rayner (2001) propose that the fact that cointegration is found between the spot and forward rate of a given foreign currency reflects the existence of arbitrage hedging of interest rates, and doesn't necessarily imply market efficiency. This theory of arbitrage in interest rates is based on the fact the real expected rate of return on capital is the same in different countries, and therefore capital moves from low-return countries to countries with high returns. The methodology used in this research has a connection with the formulation of two models, one bivariate that includes the spot rate of a foreign currency and its lagged forward rate in a period, both analyzed with Johansen's tests, as well as with those of Brenner and Kroner (1995). The other model is multivariate, since furthermore it considers the differential variable of interest rates (also lagged in a period). For this analysis it considers daily data (from May 1984 to October 1995) for the Sterling Pound, the Yen and the German Mark, with regard to the American Dollar. This research concludes that foreign currency spot rates are not only related to the lagged forward rate of the same, but also with the lagged differentials of the corresponding interest rates.

Trapletti *et al* (2002) present a cointegration analysis for three pairs of currencies, being one of the few studies, which use high frequency data. Testing the market efficiency hypothesis in short time horizons for spot rates for the Japanese Yen, the German Mark and the American Dollar. The data take into account prices to the hour from October 1st, 1992 until September 30th, 1993, which are calculated as the average among the trading prices of each of the rates. Upon performing a stationary analysis it was established that the integration order of each one of the time series involved in the study showed unit roots. On the other hand, it is important to point out that, by means of the cointegration tests of Trace and Proper Value of Johansen strong evidence of cointegration among the currencies was obtained.

Trapletti *et al* (2002) carry out, moreover, an analysis of the predictive ability of the different econometric models, fundamentally VAR models and Martingala processes. This exercise is carried out with the second half of the data and evaluating the predictions on the basis of the average square of the prediction error (MSPE) and the absolute mean of the prediction error (MAPE). On comparing the predictions performed with VAR models and Martingala processes, it is seen that the first is more efficient due to the fact that lesser prediction errors are shown. The main findings of this research reject equally the hypothesis of market efficiency, at least in its weak form. Even so, a sturdier version of the EHM that takes into account transaction costs cannot be completely accepted. This study represents a clear reference for the analysis of this research, and is highly considered in the present methodological proposal.

Wickremasinghe (2004) examines the weak and semi strong forms of the efficient market hypothesis (EHM), using in his analysis the currency of Sri Lanka as a starting point for six international currencies. In order to examine the weak form he considers the traditional unit root tests, while in order to contrast the semi strong efficiency form he utilizes the methodology of Engle and Granger, in which the tests of ADF and of Phillips-Perron (PP) are applied over the residuals of the cointegration equations. On the other hand, it carries out Johansen's tests, Granger's of causality and an analysis of the variance decomposition. In this study monthly spot rates are used for the Japanese Yen, the Sterling Pound, the American Dollar, the French Franc, the Rupee of India and the German Mark, relative to the Rupee of

Sri Lanka, for the period of January 1986 until November 2000. The principal results point to the fact that evidence exists for rejecting the semi strong version of the EHM, since by means of the cointegration tests used it is possible to assert that some currencies have a long term dependency, and, what is more, a relationship of causality as in the case of the American Dollar and the Rupee of India, and of the Yen with the Sterling Pound.

In general it should be pointed out that the majority of the afore-mentioned publications use daily, monthly, or quarterly data in their methodologies, and that is exactly the main factor common in all studies related to the analysis of cointegration among foreign currency rates, whether they be spot or forward. It is important to point out that, on utilizing that methodology there does not exist any alignment of the data used, in the sense that exchange markets have different closing times in different countries throughout the world. This is complemented by the fact that the treatment given to the prices in the different databases is diverse. In some databases the daily price could reflect the average value of the day while in others the closing value in the corresponding market is considered. Therefore, inconsistencies could easily show up in the data, which in turn could lead to inaccurate results.

Ten years ago, available data had, at the very best, a daily time horizon, which meant that 20 years of data could be reflected in about 7300 observations. Nowadays data is available with a click, that is, at the transaction level. That is why today, 20 years of data could amount to billions of observations, counting in this way on the possibility of approaching analysis in a much more complete and realistic manner.

In this way Engle and Russell (2004) check the high frequency financial data available nowadays in the different existing databases, analyzing the characteristics of this data (irregularity, variability, patterns, and time dependency), as well as the econometric framework behind, stressing new and interesting challenges for researchers[5].

This research methodology uses foreign currency data aligned to the minute, that is to say, exchange rates aligned with respect to the day, hour, and minute of another exchange pair to be analyzed. For example, information is presented with regard to the Danish Crown for the day of May 10, 2005, at 15:00 hours, as is the respective price of the Euro for the same above-mentioned date and time (e.g. for 15:01, 15:02 and so on)[6].

In this way, the present study looks to confirm the hypothesis of the efficiency of the European exchange market after the entrance of the Euro, using in the analysis high frequency data for spot rates of countries which have not incorporated the Euro as their sole currency, and so carry out the tests of cointegration, and the corresponding models of autoregressive vectors (VAR) with a more short-term time horizon (months and weeks).

[5] Just as Trapeltti, Geyer and Leisch (2002), Brooks and Hinich (2004) have done, among others.
[6] In order to be certain that the data represent the same time, EST (Eastern Standard Time) of New York is used.

2.- Methodology

Fama (1970) established that in efficient markets the difference between the real and expected price, based on given information, is zero, which can be mathematically described in the following manner:

$$E\left(\tilde{p}_{j,t+1}\left|\phi_t\right.\right)=\left[1+E\left(\tilde{r}_{j,t+1}\left|\phi_t\right.\right)\right]\cdot p_{j,t} \qquad (1)$$

Where E is the operator of Expected Value; $p_{j,t}$ is the price of the instrument j in the period t; $p_{j,t+1}$ is the price of the instrument j in the period t+1; $r_{j,t+1\ i}$ is the return percentage $(p_{j,t+1}- p_{j,t})/ p_{j,t}$ of an instrument j in a period t; Φ_t is a general notation for representing any set of information completely reflected in the period price t[7].

In this way, one could point out that a *Random Walk* is a special case of a Markov process, where only the current information of a variable is relevant for predicting the future. And in this way, the historical data of a variable and the way in which the present has manifested itself from the past has no importance whatsoever.

A direct application for verifying the performance of the EHM, is considering the case of the foreign currency market at the international level. This is how, in an efficient speculative market, the prices of the exchange pairs must completely reflect the information available in the market, and therefore it would be practically impossible for investors to obtain excessive returns due to speculation.

In the case of the foreign currency market[8] (S), many studies conclude that there does not exist any stable relationship between S_t and S_{t-1}. In fact, numerous research studies have concluded that foreign currency pairs change approximately as a random walk. In this way, exchange pairs can follow a random walk only if they are found in a efficient market. Given this, it must be mentioned that the predictions of foreign currency prices will not result if market efficiency theories are upheld [9].

On the other hand, with respect to the analysis of time series (ST), it must be pointed out that there are two types of ST, those that are stationary and those that are non-stationary. It is said that a stochastic process[10] is stable if its average and its variance are constant in time, and if the covariance value between two periods depends solely on the distance (lag) between these two periods and not on the time in which it has been calculated, that is, the average, the variance, and the covariance are constant with respect to time. A stable ST will tend to return to its average and fluctuations around this average will have a constant range[11].

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[7] The accent (˜) indicates that $p_{j,t+1}$ and $r_{j,t+1}$ are forecast variables.
[8] The foreign exchange market is characterized by S_t, where t represents the time period in question.
[9] These kinds of markets are characterized because the arrival of new information (relevant for price adjustment in markets) is random, and because an extensive number of participants will have equal access to this new information, so each of the investors will commercialize with the same information, and in this way prices will adjust themselves. This description would characterize foreign currency markets according to Bishop and Dixon (1992).
[10] A stochastic process is a collection of random variables ordered in time.
[11] The fulfillment of these conditions assures that $\{Y_t\}$ is weakly stationary.

The majority of ST show strong tendencies, so if a regression is performed between two (or more) unstable variables I(1)[12], it could be mistakenly concluded that there is a strong relationship between them. Suppose that one has first order integrated series I(1) $Y_t=Y_{t-1}+\mu_t$ y $X_t=X_{t-1}+\nu_t$, and the regression of Y_t over X_t is performed, even when there does not exist a strong relationship between both series, it's possible to obtain statistics t very significant and a very high R^2, since the increasing tendencies of both variables could indicate a strong statistical relationship between both series, but such a correlation would be spurious.

Granger (1986) points out that if a currency pair is found in an efficient market, then the price series of these foreign currencies cannot be cointegrated. If (X_t,Y_t) are a pair of currency prices in an efficient speculative market, they cannot be cointegrated, because if they were one price could be used to predict the price of the other, which contradicts the EHM[13].

This study will verify the existence of a long term relationship among the different foreign currencies (the Danish Crown, The Sterling Pound, the Swiss Franc and the Norwegian Crown) and the Euro (all against the American Dollar). The first test to be used to check cointegration is that called the Engle and Granger test, which contrasts the existence of stationarity of the residuals of a cointegration regression. To do this the ADF test will be used over the errors of this equation (see equation 3), considering the critical values tabulated by MacKinnon (1991). The second test to be used to confirm the existence of cointegration among foreign currencies is that called the Johansen test, which takes into account autoregressive vector models (VAR) for its analysis. The procedures of both tests will be described as follows.

- **Test of Engle and Granger**

The procedure for verifying the existence of cointegration using this test is detailed as follows:

1. Suppose that one has two time series $\{Y_t\}$ y $\{X_t\}$, and a regression is made of Y over X in the following manner:

$$Y_t = \beta_1 + \beta_2 \cdot X_t + \mu_t \qquad (2)$$

2. Now the residual is expressed as a linear combination of the series, that is to say, it is formulated as:

$$\mu_t = Y_t - \beta_1 - \beta_2 \cdot X_t \qquad (3)$$

3. Finally the ADF test must be applied to the values of μ_t. If it is discovered that the series of residuals is stable, that is I(0), then it could be concluded that these two variables (X_t, Y_t) are cointegrated.

[12] It is said that a series is integrated in order d, this is I(d), if it becomes stationary after differentiating it d times.
[13] A complete review of the hypothesis of market efficiency can be found in "The Efficient Market Hypothesis: A Survey", study by Beechey, Gruen and Vickery.

- **Test of Johansen**

Concerning the test of Johansen, the regression to be used is the following:

$$\Delta Y_t = \alpha + \Pi \cdot Y_{t-1} + \sum_{J=1}^{k} \Gamma_j \cdot \Delta Y_{t-j} + \mu_t \qquad (4)$$

Where Y_t is a vector of n variables, α is a vector of n constants, Π and Γ_j (j=1,…, k) are matrices of order parameters n x n, and μ_t is the vector of n residuals. There is cointegration if the matrix Π has a range greater than zero, that is the null hypothesis (H_0) is that the range(Π)=h, and the alternative (H_1) is that the range(Π)=n, where h represents the number of cointegration vectors. Under the alternative hypothesis, the range of Π is n, which could only happen if the n variables contained in the vector Y_t are stable, that is I(0).

- **Autoregressive Vector Models (VAR)**

Finally, as was previously shown, the prediction experiment will look to contrast two types of Var models (weekly and monthly) for those foreign currencies that have significant long term relationships. The best model will be chosen according to which has fewer *Absolute Prediction Errors*, that is to say, if one has a real value Y_t and a predicted value $\widetilde{Y}_t$, then that model will be selected which has a lesser $\left| Y_t - \widetilde{Y}_t \right|$ average.

The VAR methodology is a generalization of the univariate autoregressive models, since it takes into account a system of regressions composed of endogenous variables[14] explained by its values lagged over time.

For example, if two variables X and Y are assumed, where Y affects X and this last also influences Y, one would have an ideal situation for the application of a VAR model in the following way:

$$Y_t = \alpha + \sum_{j=1}^{k} \beta_j \cdot Y_{t-j} + \sum_{j=1}^{k} \gamma_j \cdot X_{t-j} + \mu_t \qquad (5)$$

$$X_t = \alpha' + \sum_{j=1}^{k} \theta_j \cdot Y_{t-j} + \sum_{j=1}^{k} \gamma_j \cdot X_{t-j} + \mu'_t \qquad (6)$$

Where μ are the terms of error, also called impulses, innovations, or shocks; and k is the length of the lag to be used.

[14] Usually exogenous variables are not incorporated in the model.

3.- Results of the Cointegration Analysis

- **Concerning the Data**

This research uses extremely separated data, with time intervals of one minute between datum and datum, and therefore has implications in the methodology to be used and in the results to be expected.

The foreign currencies analyzed in this research are measured in the same time zone (EST), in order to be certain that the prices represent exactly the same time period, and at the same time use data, which is completely aligned. Said data has been obtained from the e-signal database, which provides information in real time from financial markets (whether they be foreign currencies, stocks, price indexes, etc.) around the world. The analysis period considers foreign currency spot prices from May 10th to August 10th of 2005.

One important step for this analysis relates to exposing stylized facts, as is the descriptive statistic of the prices and volume bought and sold of each of the currencies involved in this study, that is to say, for the Danish Crown (USD/DKK), the Sterling Pound (GBP/USD), the Norwegian Crown (USD/NOK), the Swiss Franc (USD/CHF) and the Euro (EUR/USD), which is presented in the appendix of this study[15].

- **Augmented Dickey-Fuller Test Over the Residuals (ADF)**

The ADF test requires that a dependent variable in the cointegration equation be selected. The test results are not immune to such a choice. However, theoretical foundations can guarantee the choice of a certain causal relationship. In particular, the Euro was chosen in all cases as the independent variable, due to the fact that it is not logical to think that, for example, the Danish Crown "moves" the Euro. That is to say, in this way it is possible to contrast if, effectively, the price of these foreign currencies (Swiss Franc, Danish Crown, Norwegian Crown and Sterling Pound) is neutral to increases (or decreases) in the spot rate of the Euro.

In the following, the ADF test results for the complete series are presented, with the aim of verifying the presence of a long-term relationship among the foreign currencies in question.

Table 1: Results of ADF Over the Residuals[16]

Cointegration Equations	ADF Test
Ln(Euro) and Ln(DKK)	-8,750**
Ln(Euro) and Ln(NOK)	-2,451*
Ln(Euro) and Ln(CHF)	-1,759
Ln(Euro) and Ln(GBP)	-1,047

*Note: *(**) represent test statistics significant to 5%(1%).*

[15] Moreover in the appendix the results of the ADF stationarity tests are presented on each of the series, from which it is concluded that all present unit roots, that is, they are I(1).

[16] Cointegration was tested using the cointegration equations with the complete series of prices, that is, data to the minute from May 10 to August 10, 2005.

In each of the tests 5 lags were included, and the cointegration equations of each of the cases incorporated as many lags as the Akaike Information Criteria[17] (AIC) determined. The following will set forth the results obtained in those cases where high volumes of these foreign currencies are traded on the international market[18].

Table 2: Cointegration Results for High Volumes Traded

Cointegration Equation	ADF Tests
Ln(Euro) and Ln(DKK)	-3,772**
Ln(Euro) and Ln(NOK)	-2,482*
Ln(Euro) and Ln(CHF)	-1,172
Ln(Euro) and Ln(GBP)	-0,862

*Note: *(**) represent test statistics significant to 5%(1%).*

With respect to the objective of establishing time criteria to prove the cointegration hypothesis, it must be pointed out that ADF tests were performed at the following times: 8:40 to 9:30 , and from 9:50 to 10:40 according to New York time (Eastern Standard Time).
The results for each one of the foreign currencies is showed as follows[19]:

Table 3: Cointegration Results for Specific Times

Cointegration Equation	ADF Test	
	Time 8:40 to 9:30	Time 9:50 to 10:40
Ln(Euro) and Ln(DKK)	-2,092*	-1,673
Ln(Euro) and Ln(NOK)	-1,951*	-1,878
Ln(Euro) and Ln(CHF)	-1,037	-0,950
Ln(Euro) and Ln(GBP)	-0,903	-1,035

*Note: *(**) represent test statistics significant to 5%(1%).*

On observing the ADF values for each of the cointegration equations shown in the previous tables it can be concluded that the foreign currencies which cointegrate with the Euro (EUR/USD) are the Danish Crown (USD/DKK) and the Norwegian Crown (USD/NOK), but it cannot be asserted that the degree of cointegration increases when there is a high demand for these foreign currencies on the international market, not even when one considers the volumes traded from 8:40 to 9:30, and from 9:50 to 10:40. Since even ADF values diminish when cointegration is proved in these two situations.

[17] The problem of how many lags to include implies specification of the model. The criteria AIC selects additional lags until the coefficient of determination (R^2) stops increasing.
[18] Was considered a high volume, when the transactions exceeded by 80% the average of the volumes traded for each of the currencies.
[19] On the other hand, it must be pointed out that the critical values (CV) used are not those estimated by Dickey-Fuller, but the appropriate CV have been tabulated by MacKinnon (1991).

- **Test of Johansen**

As an alternate way of testing the existence of cointegration among the foreign currencies studied in this research, the trace test proposed by Johansen (1985) will be performed. This methodology offers a way of determining the number of cointegration vectors, their identification and their manner of inclusion in a general estimation.

Basically, in this section, bivariate models will be set out for spot rates for the Danish Crown (USD/DKK), the Norwegian Crown (USD/NOK), the Swiss Franc (USD/CHF) and the Sterling Pound (GBP/USD), each one versus the price of the Euro (EUR/USD). In accordance with the Johansen's trace test, a relationship of cointegration would exist when H_o is not rejected. At the most a cointegration relationship exists; versus the alternative H_1: There are two cointegration relationships (This can only happen if the two variables contained are stable, that is I(0)).

In the following, the results of Johansen's test for the complete series (the whole period) are presented for each of the foreign currencies analyzed.

Table 4: Results for the Test of Johansen for Complete Series

Currencies	Valor Propio	Trace Test	Critical Value to 5%	Critical Value to 1%	Number of Cointegration Relationships(Ho)
EUR/USD v/s USD/DKK	0,002699	261,218	15,410	20,040	None **
	6,45E-05	6,088	3,760	6,650	1 at the most *
EUR/USD v/s USD/NOK	0,000112	16,661	15,410	20,040	None *
	6,42E-05	6,072	3,760	6,650	1 at the most *
EUR/USD v/s USD/CHF	9,53E-05	15,371	15,410	20,040	None
	6,20E-05	6,061	3,760	6,650	1 at the most *
EUR/USD v/s GBP/USD	7,46E-05	10,114	15,410	20,040	None
	2,90E-05	2,829	3,760	6,650	1 at the most

*Note: *(**) indicate the rejection of H_o to 5%(1%) significance.*

It is relevant to point out that the test assumes the presence of deterministic tendencies in the series, and two lags were used in the VAR. This condition will be used in all the tests carried out, since according the Akaike Criteria (AIC) it wasn't ideal to include more lagged variables.

In Table 4 it can be appreciated that the test solely indicates the presence of a relationship of cointegration between the Danish Crown (USD/DKK) and the Euro (EUR/USD), with a reliability level of 99%, which is given by[20]:

$$\underset{\quad (59186)}{\text{Ln}(\text{USD}/\text{DKK}) = 2{,}0128} \underset{(-6127)}{- 1{,}0241 \cdot \text{Ln}(\text{EUR}/\text{USD})} \qquad (7)$$

R^2=0,9975
n=94389

The following will set forth the results obtained from Johansen's trace test when high volumes of these currencies are traded on the international market[21].

Once again it should be pointed out that the test assumes the presence of deterministic tendencies in each of the series to be studied, and that two lags in the VAR were used.

Table 5: Results for the Test of Johansen when High Volumes of these Currencies are Traded on the International Market

Currencies	Proper Value	Trace Test	Critical Value to 5%	Critical Value to 1%	Number of Cointegration Relationships (Ho)
EUR/USD v/s USD/DKK	0,004436	58,640	15,410	20,040	None **
	9,86E-04	10,652	3,760	6,650	1 at the most **
EUR/USD v/s USD/NOK	0,001427	20,416	15,410	20,040	None **
	3,83E-04	4,319	3,760	6,650	1 at the most *
EUR/USD v/s USD/CHF	0,000954	16,495	15,410	20,040	None *
	3,00E-04	3,948	3,760	6,650	1 at the most *
EUR/USD v/s GBP/USD	7,73E-04	13,529	15,410	20,040	None
	1,27E-04	1,907	3,760	6,650	1 at the most

*Note: *(**) indicates the rejection of H_o to 5%(1%) significance*

[20] The values in parentheses are the statistics t of each of the coefficients.
[21] Was considered a high volume according to the same criteria used in the ADF tests, that is, when the transactions exceeded by 80% the average of the volumes traded for each of the currencies.

From Table 5 it can be concluded that it is not possible to reject, at 1% significance, the hypothesis that at the most there does exist a relationship of cointegration between the Norwegian Crown (USD/NOK) and the Euro (EUR/USD), according to the condition of high volumes traded internationally, therefore a long term dependency would exist between these two currencies.

The afore-mentioned can be represented through the following cointegration equation[22]:

$$\mathrm{Ln}(\mathrm{USD}/\mathrm{NOK}) = 2{,}0273 - 0{,}7848 \cdot \mathrm{Ln}(\mathrm{EUR}/\mathrm{USD}) \qquad (8)$$

(-201,5) (2638,6)

R^2=0,7826
n=11278

Finally, the hypothesis of cointegration at certain times of day will be contrasted, just as was done with the ADF tests, the proposed times are from 8:40 to 9:30, and from 9:50 to 10:40 (Eastern Standard Time). Results are shown in the following:

Table 6: Results of the Test of Johansen when Currencies are Traded at Certain Specific Times

Currencies	Time	Proper Value	Trace Test	Critical Value to 5%	Critical Value to 1%	Number of Cointegration Relationships (Ho)
Danish Crown v/s Euro	8:40 - 9:30	0,004805	22,526	15,410	20,040	None **
		1,78E-03	6,082	3,760	6,650	1 at the most *
	9:50 – 10:40	0,004727	21,944	15,410	20,040	None **
		1,69E-03	5,766	3,760	6,650	1 at the most *
Norwegian Crown v/s Euro	8:40 - 9:30	0,002518	12,746	15,410	20,040	None
		1,21E-03	4,140	3,760	6,650	1 at the most *
	9:50 - 10:40	0,002041	10,490	15,410	20,040	None
		1,03E-03	3,516	3,760	6,650	1 at the most
Swiss Franc v/s Euro	8:40 - 9:30	2,16E-03	10,607	15,410	20,040	None
		9,47E-04	3,235	3,760	6,650	1 at the most
	9:50 - 10:40	2,36E-03	11,478	15,410	20,040	None
		1,00E-03	3,429	3,760	6,650	1 at the most
Sterling Pound v/s Euro	8:40 - 9:30	2,37E-03	10,104	15,410	20,040	None
		6,25E-04	2,107	3,760	6,650	1 at the most
	9:50 - 10:40	2,65E-03	11,164	15,410	20,040	None
		6,20E-04	2,117	3,760	6,650	1 at the most

*Note: *(**) indicates the rejection of H_o to 5%(1%) significance.*

[22] The values in parentheses are the statistics t of each of the coefficients.

From Table 6 the existence of cointegration between the Danish Crown and the Euro can be appreciated, for both time periods studied, that is to say, there is a long term relationship for these two pairs from 8:40 to 9:30 and from 9:50 to 10:40 with a reliability level of 99%.

The equations that relate these two currencies at those specific times are the following:

$$\underset{\substack{(11356)\\ R^2=0,9975\\ n=3417}}{\underset{(-1174)}{\mathrm{Ln}(USD/DKK) = 2,0126 - 1,0228 \cdot \mathrm{Ln}(EUR/USD)}} \qquad (9)$$

$$\underset{\substack{(11274)\\ R^2=0,9975\\ n=3417}}{\underset{(-1165)}{\mathrm{Ln}(USD/DKK) = 2,0128 - 1,0237 \cdot \mathrm{Ln}(EUR/USD)}} \qquad (10)$$

In a general way, it can be concluded that there exists a significant relationship of cointegration between the Danish Crown and the Norwegian Crown versus the Euro. For this reason, by way of complementing this research, a prediction experiment will be performed for the Danish Crown and another for the Norwegian Crown, using the complete series of prices, since if it is true that when high volumes were traded on the international market and/or cointegration was tested at certain specific times evidence was found of a long term relationship, stronger relationships were obtained when the complete series of prices were contrasted[23].

4.- VAR Models

In this section different VAR models are set out, as a way of analyzing the predictive ability of each model for the spot rate of the Danish Crown (USD/NOK) versus the Euro (EUR/USD), given the relationships of cointegration found in the previous sections. The data used in this predictive exercise is considered from October 2 until December 30, 2005.

Six different models will be carried out, using data with a frequency of 10, 30 and 60 minutes and with time horizons of 1 and 2 months of data in the models. And the quality of the predictions will be compared using the criteria of the Root Mean Square Error (RMSE) and of the Mean Absolute Error (MAE), in order to know which model best explains the performance of the spot rate of the Danish Crown.

It is relevant to point out that data from October and November will be used in the models, in order to forecast prices for December, and so count on a base for comparison.

[23] In the Appendix of this study the graphs of the series of prices of the Danish Crown, the Norwegian Crown and the Euro are given, in order to have a complementary view of the cointegration results found.

It is important to mention that the VAR models to be estimated are in the following form:

$$LN(DKK)_t = \theta_o + \sum_{i=1}^{k} \theta_i \cdot LN(DKK)_{t-i} + \sum_{i=1}^{k} \phi_i \cdot LN(EUR)_{t-j} + \nu_t \quad (11)$$

Where k is the number of lags included, θ and ϕ are the coefficients of the model and ν_t is the term of error.

As follows, Table 7 shows the number of lags included in the VAR, selected using the Schwarz Criteria (SC):

Table 7: Lags Included according to the Schwarz Criteria

MODEL	Number of Lags in the VAR (K)							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
I-A	-19,1008	**-19,2278***	-19,1943	-19,1768	-19,1363	-19,1049	-19,0561	-19,0178
I-B	-19,2427	**-19,3388***	-19,3329	-19,3382	-19,3231	-19,3059	-19,2981	-19,2733
II-A	-19,9321	-20,0586	-20,0990	**-20,1115***	-20,1030	-20,0802	-20,0594	-20,0397
II-B	-20,0383	-20,1477	-20,1871	-20,2130	**-20,2181***	-20,2090	-20,1984	-20,1943
III-A	-21,2339	-21,3930	-21,4501	-21,4880	-21,5009	**-21,5163***	-21,5122	-21,5038
III-B	-21,2916	-21,4386	-21,5006	-21,5371	-21,5515	-21,5705	**-21,5740***	-21,5737

*Note: Models I, II and III use data with a frequency of 60, 30 and 10 minutes respectively, while A and B are interpreted as the incorporation of 1and 2 months of data in the VAR models. *Minimum.*

On the other hand, the results of the forecasts made with the different models formulated are shown in the following:

Table 8: Comparison of the Forecast Models

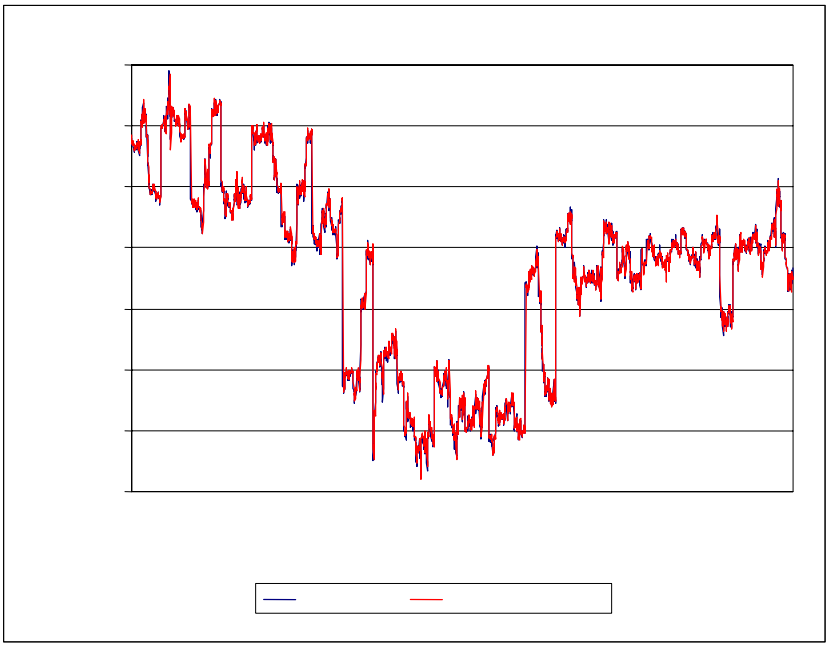
MODEL	Frequency of Data (Minutes)	Data Incorporated (Months)	RMSE ($\cdot 10^{-2}$)	MAPE ($\cdot 10^{-2}$)
I-A	60	1	0,1966	0,1205
I-B		2	0,1841	0,1007
II-A	30	1	3,2102	3,2042
II-B		2	1,4116	1,4043
III-A	10	1	0,0786	0,0380
III-B		2	**0,0774***	**0,0358***

*Note: * Minimum*

In Table 8 it can be seen clearly that the best model for performing forecasts is that which incorporates more information, that is two months of historical prices (October and November), and that which uses data with a frequency of 10 minutes.

In Graph 1 the series of real prices (Blue Series) and forecasted (Red Series) for the Danish Crown can be appreciated, using the Model III-B (2 months of data with a frequency of 10 minutes).

Graph 1: Real Prices vs. Forecasted of the Danish Crown for December 2005



In Graph 1 the quality of the forecast using this type of model is clearly seen, confirming the invalidity of the hypothesis of market efficiency for these foreign currency markets, and the concrete possibility of forecasting the value of the Danish Crown (USD/DKK), using two months of data for the Euro (EUR/USD) with a frequency of 10 minutes.

The results of the VAR model (III-B) are shown in the following:

Table 15: Coefficients Estimated for Model III-B

VARIABLES	COEFFICIENT	STATISTIC T
$Ln(DKK)_{t-1}$	0,30976	3,29471
$Ln(DKK)_{t-2}$	0,10383	-1,20569
$Ln(DKK)_{t-3}$	0,10606	-0,47888
$Ln(DKK)_{t-4}$	0,08710	-0,97866
$Ln(DKK)_{t-5}$	0,10649	0,28465
$Ln(DKK)_{t-6}$	0,15772	0,48907
$Ln(DKK)_{t-7}$	0,03701	-1,19525
$Ln(EUR)_{t-1}$	0,68953	20,60381
$Ln(EUR)_{t-2}$	-0,09850	1,43791
$Ln(EUR)_{t-3}$	-0,10635	0,50346
$Ln(EUR)_{t-4}$	-0,08469	1,01518
$Ln(EUR)_{t-5}$	-0,12487	-0,80898
$Ln(EUR)_{t-6}$	-0,14567	-0,24537
$Ln(EUR)_{t-7}$	-0,04052	1,35843
Constant	0,18438	-0,25445
R^2	0,9951	

5.- Conclusions

The main objective of this study was to develop the problem of efficiency European foreign currency markets after the entrance of the Euro, using econometric techniques allowing for the contrast of the existence of a long term relationship between the spot rate of the Euro (EUR/USD) and other European currencies. Before proceeding with the tests of cointegration for the foreign currencies in question, a stationarity analysis was carried out by means of ADF tests for each one of the series, demonstrating in all cases the existence of unit root, which coincides with classic studies showing that the movement of spot rates of foreign currencies over time, are non-stationary processes.

By means of the cointegration tests of Johansen and ADF, the significant existence of cointegration between the Danish Crown and the Euro, and the Norwegian Crown and the Euro is concluded. On the other hand, cointegration tests were carried out supporting the hypothesis that there exists a common long term relationship between exchange pairs when high volumes of said pairs are traded on the international market, obtaining as main results the existence of cointegration among the same afore-mentioned foreign currencies. Finally, cointegration tests were performed for certain specific times of day, concluding the presence of cointegration for the same currencies between 8:40 and 9:30, and between 9:50 and 10:40. (Eastern Standard Time).

With respect to this last, it is important to point out that although when cointegration was tested at certain specific times or when there were high volumes traded on the international market, and evidence was found of a long term relationship among exchange pairs, stronger relationships were obtained when complete price series were contrasted. For this reason it is not possible to state that there is an increase in the integration of these variables when a greater demand at the international level exists.

It is relevant to point out that economic policy decisions, whether they are fiscal or monetary, of countries such as Denmark[24], depend directly and indirectly on the performance of the Euro. In fact, the strong correlation (0,95 with data to the minute and 0,99 with daily data) and cointegration existing between the Danish Crown (USD/DKK) and the Euro (EUR/USD) can be explained as a consequence of the exchange policy adopted by the Danish Central Bank (Danmarks Nationalbank), since the Danish Crown is strongly linked to the Euro because of the ERM II (Exchange Rate Mechanism II), which has the aim of assuring stability between the Euro and the currencies of those countries that have not adopted the Euro as their sole currency. Participation in this mechanism is voluntary and, since January 2001, Denmark is the only country which uses it.

Basically the objective of the Danish Central Bank's exchange policy is to maintain a stable relationship between the Danish Crown and the Euro, an objective which is based on a band of prices, which in the case of Denmark fluctuate in ± 2,25%, and which is maintained via

[24] That while being part of the European Union, have not adopted the Euro as their official currency.

direct intervention by the monetary authority, that is, the trading of foreign currencies in the exchange market, and adjustment of interest rates.

Moreover, it is also important to point out that, as previously mentioned, the correlation between these two foreign currencies is very close to one (0,99) when daily prices are considered, but drops a little (0,95) when taking into account data to the minute, which implies that using this methodology the possibility of short term arbitrage would exist (Intraday) since the adjustments performed by the Danish Central Bank are not instantaneous and have a small time lag. In this way, in an inefficient foreign currency market, government authorities must determine the best way of intervening in the exchange rates, in order to reduce the volatility of the exchange series.

In section 4 of this research 6 different forecasting models were presented, using time horizons of 1 and 2 months, with frequencies of 60, 30 and 10 minutes in the data. It is in this way that the best VAR model, according to the minimum RMSE and MAE criteria, is that which incorporated greater market information (2 months) and with a frequency of 10 minutes between datum and datum, achieving with this a great forecasting ability for the price of the Danish Crown in function of the Euro, in this way permitting the investors to devise arbitrage strategies with these two pairs of foreign currencies.

Finally, it is important to point out that an alternative approach for proving market efficiency has a connection with the existence of cointegration between a currency's spot rate and its corresponding forward rate, where the coefficient of cointegration must be equal to one. This hypothesis of efficiency implies that the forward rate and the spot exchange rate of a given foreign currency move each other simultaneously. For the former, it is recommended that future studies be able to test the presence of cointegration between the spot rate of a currency such as the Danish Crown and its corresponding forward rate, using the same methodology employed in this study as a way of capturing misalignments that can be produced between these two financial instruments, and in this way dynamically model these imbalances using high frequency data. Counting in this way with another approach that proves the inefficiency of markets with certain financial assets, and therefore the possibility exists of obtaining higher than normal returns due to arbitrage.

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Appendix. Descriptive Statistics, Correlation Analysis, Tests of Stationarity

Here the descriptive statistic of the prices and volumes traded of each of the pairs in the study:

Table 9: Descriptive Statistics of Foreign Currency Prices and Volumes

	Danish Crown		Sterling Pound		Norwegian Crown		Swiss Franc	
	Price	**Volume**	**Price**	**Volume**	**Price**	**Volume**	**Price**	**Volume**
Observations	94389	94389	94389	94389	94580	94580	97701	97701
Mean	6,084	21,964	6,084	21,964	6,485	26,914	1,264	70,889
Median	6,120	19,000	6,120	19,000	6,472	24,000	1,270	58,000
Maximum	6,277	350,000	6,277	350,000	6,702	253,000	1,308	526,000
Minimum	5,765	1,000	5,765	1,000	6,259	1,000	1,196	1,000
Standard Deviation	0,119	14,889	0,119	14,889	0,090	17,115	0,027	54,106
Kurtosis	2,609	9,200	2,609	9,200	2,188	4,827	2,252	6,652

It is important to mention that as each of these currencies is perfectly aligned to the Euro (EUR/USD), the descriptive statistic of this currency varies slightly for each of the cases in the study. On the other hand, the descriptive statistic of the Euro is prepared as follows:

Table 10: Descriptive Statistic of Prices and Volumes of the Euro

	Euro	
	Price	**Volume**
Observations	94389	94389
Mean	1,22474	70,98734
Median	1,21760	60,00000
Maximum	1,29110	451,00000
Minimum	1,18680	1,00000
Standard Deviation	0,023811	49,31607
Kurtosis	2,71024	6,41066

Finally, by way of reference, a matrix of correlations with the five foreign currencies involved in the study is shown in Table 11: the Euro, the Swiss Franc, the Danish Crown, the Sterling Pound and the Swiss Franc, measured in American Dollars[25].

[25] To make this matrix of correlations original data to the minute obtained through e-Signal were considered.

Table 11: Matrix of Correlations of the Currencies Spot Rates

	Euro	Swiss Franc	Danish Crown	Sterling Pound	Norwegian Crown
Euro	1				
Swiss Franc	-0,961	1			
Danish Crown	-0,949	0,910	1		
Sterling Pound	0,742	-0,874	-0,688	1	
Norwegian Crown	-0,776	0,772	0,861	-0,591	1

In Table 11 the important level of correlation of the Euro with the Swiss Franc and the Danish Crown can be appreciated. The Sterling Pound and the Norwegian Crown present an important but lower level of correlation with the Euro.

- **Tests of Stationarity**

In this study formal tests of stationarity for each of the variables and their respective first differences were performed. In the following a table is presented with the results of the ADF test for each of the time series and their first differences:

Table 12: Results of ADF Test

Variable	ADF STATISTIC	
	At Level	In First Differences
USD/DKK	-1,483703	-154,8188**
USD/NOK	-1,96781	-158,2047**
USD/CHF	-1,260702	-151,1548**
GBP/USD	-1,27123	-149,178**
EUR/USD	-1,479827	-153,1881**

*Note: ** indicates a level of significance at 1% according to the t-student statistics.*

From Table 12 it can be concluded that in all cases (that is, USD/DKK, USD/NOK, USD/CHF, GBP/USD and EUR/USD) it is not possible to reject the null hypothesis, and therefore it can be affirmed with a certainty of 99% that one is in the presence of unit root. From this analysis it is rigorously concluded that the five series are I(1).